Diffo Equ°. – Assignment 02

Equations Reducible to VSF

Q.1. Solve the following diff. eqns

(a) $\cos(x+y)\,dy = dx$ [Sol. $\tan\frac{x+y}{2} = y+c$]

(b) $\frac{dy}{dx} = \cos^2(x+y)$ [Sol. $y = x + \frac{1}{2}\sin(2x+2y)+c$]

(c) $(x+y)^2 \cdot \frac{dy}{dx} = 1$; when $y=0, x=1$ [Sol. $y - \tan^{-1}(x+y) + \pi/4 = 0$]

(d) $\frac{dy}{dx} = (4x+y+1)^2$ [Sol. $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x+c$]

Homogeneous Diffo Equations

Q.2 Solve the following diff eqns

(a) $(3xy+y^2)\,dx - (x^2+xy)\,dy = 0$ [$\log|y| + y/x = 3\log|x| + c$]

(b) $dy/dx = (x+y)/(x-y)$ [$\tan^{-1} y/x = \frac{1}{2}\log(x^2+y^2)+c$]

(c) $(x^2-y^2)\,dx + 2xy\,dy = 0$ [$x^2+y^2 = cx$]

(d) $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [$\sin y/x = cx$]

(e) $(x^3+y^3)\,dy - x^2y\,dx = 0$ [$y = c\cdot e^{x^3/3y^3}$]

(f) $y^2 + x^2\frac{dy}{dx} = xy\cdot\frac{dy}{dx}$ [$y/x - \log|y| = 1$]

given $y=1$ when $x=1$.

First-Order Linear Diff. Eqns

Q.3. (a) $\frac{dy}{dx} - y = x\cdot e^x$ [$y = e^x(x^2/2 + c)$]

(b) $(3y^2-x)\,dy = y\,dx$ [$xy = y^3 + c$]

(c) $4\frac{dy}{dx} + 8y = 5\cdot e^{-3x}$ [$y e^{2x} = -\frac{5}{4}e^{-x} + c$]

(d) $y' + 2y = \sin x$ [$y = \frac{1}{5}(2\sin x - \cos x) + c\cdot e^{-2x}$]

(e) $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$ [$y\log x = -\frac{2\log x}{x} - \frac{2}{x} + c$]